RISK-ADJUSTED PERFORMANCE ANALYSIS

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INTRODUCTION (1/2)

Common wisdom today: Performance is only a *biased* and *noisy* signal for the quality of asset management.

BIAS: Risk-return trade off

NOISE: Skill versus luck

Risk-adjusted performance analysis is about quantifying and analyzing unbiased performance. It can also be used to distinguish skill from luck.

This presentation wants to summarize the best practice concepts and methods in risk-adjusted performance analysis. It is of a descriptive nature.

INTRODUCTION (2/2)

CONSULTANTS SWITZERLAND

- Quantitative performance analysis as a criterion for manager selection has been practiced for about 5 years → a new toy
- Most often requested statistics: Sharpe and Information Ratio

STRUCTURED ALPHA™ (Watson Wyatt)

- **Alpha**: Net Fund Return – Net Benchmark Return.
  Net = Fees & switching costs
- **Sigma**: Tracking Error = Standard Deviation of Alpha

  → Financial Factors summarized in…
  Investment Efficiency: Net Alpha / TE = IR. Used to rank managers
- **Theta**: Non-financial factors are of importance to trustees: 'Sleep Well' factors (loss aversion), 'Seems Good' factors (brand names)
  According to WW, the relative influence of financial and Theta factors is 50/50.
  Source: 'Global Industry Survey', WW, 1999.
- There exists a trade off between financial and Theta factors. That's why you need WW's consulting service…

FRAMEWORK

1. CLIENT PREFERENCES

Client likes return, dislikes risk*:

$$U = U(\mu_P, \sigma_P)$$

$$dU = \frac{\partial U}{\partial \mu_P} d\mu_P + \frac{\partial U}{\partial \sigma_P} d\sigma_P$$

$$\frac{\partial U}{\partial \mu_P} > 0$$

$$\frac{\partial U}{\partial \sigma_P} < 0$$

*risk is usually defined as the second moment of the return distribution.

2. BENCHMARKING

- **Client** chooses benchmark and sets targets/limits for alpha, beta a.s.o. *at* inception
- **Portfolio Mgt** controls alpha, beta and beta *after* inception

3. INDEX MODELS

$$\mu_P - r_f = \alpha + \beta \cdot (\mu_B - r_f) + \varepsilon$$

Validity of index models to analyze performance largely depends on the implementation of benchmarking!

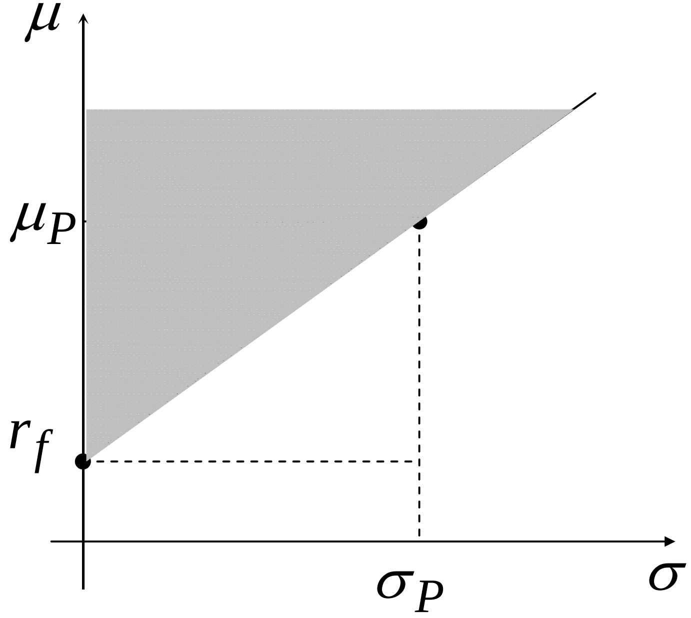
SHARPE RATIO (1/2) – DEFINITION

$$S = \frac{\mu_P - r_f}{\sigma_P}$$

μ_P ...*Portfolio Return*

r_f ...*Riskfree Rate*

σ_P ...*Portfolio Volatility*



SHARPE RATIO (2/2) - APPLICATION

MEASUREMENT

- Annualized portfolio return, portfolio volatility
- Annualized risk-free rate
  - Choice is important because it can change ranking
  - Problematic in an international context
- Aggregation
  - No straight-forward adding-up because of covariance effects between volatilities
- Are negative values ambiguous?

$$+\frac{\mu_P - r_f}{\sigma_P \downarrow} \uparrow \qquad -\frac{\mu_P - r_f}{\sigma_P \downarrow} \downarrow$$

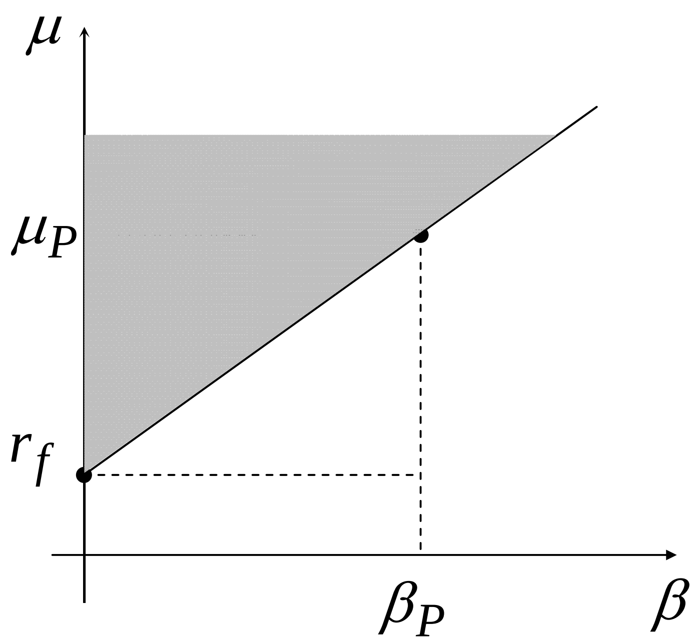
INTERPRETATION

- Summary of the first two moments of the portfolio excess return distribution. Model-free
- Suitable for comparisons across asset classes
- Target in Mean-Variance Optimization
- Does not assume a benchmark. Implicit benchmark is risk-free rate.
- Statistical hypothesis testing: test for non-zero performance

  t-Stat = S * sqrt(T)

TREYNOR RATIO (1/2) - DEFINITION

$$T = \frac{\mu_P - r_f}{\beta_P}$$



μ_P ...*Portfolio Return*

r_f ...*Riskfree Rate*

β_P ...*Portfolio Beta*

$$\beta = \frac{\sigma_{PB}}{\sigma_B^2} = \rho_{PB}\frac{\sigma_P}{\sigma_B}$$

σ_{PB} ...*Covariance*

ρ_{PB} ...*Correlation*

TREYNOR RATIO (2/2) - APPLICATION

MEASUREMENT

- Annualized portfolio return, annualized risk-free rate
- Estimation of beta can be distorted by market timing. Extensions: Squared regression, H/M regression
- Aggregation: Straight-forward. Beta of aggregate is weighted sum of constituent's betas

INTERPRETATION

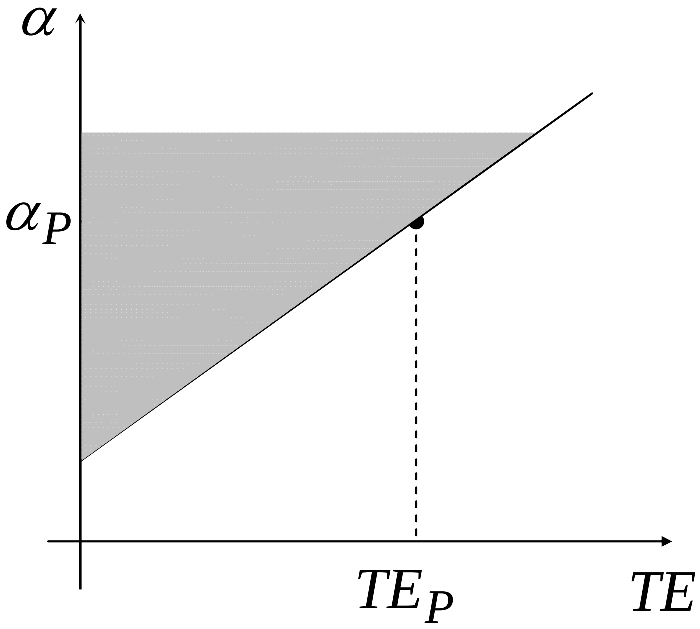
- Accounts for systematic and unsystematic risk (CAPM-based): Only systematic risk is considered.
- Comparison across different asset classes problematic (beta is dependent on benchmark)
- Choice of benchmark affects ranking

INFORMATION RATIO (1/3) - DEFINITION

$$IR = \frac{\alpha_P}{TE_P}$$

α_P ...*Portfolio Alpha*

TE_P ...*Porfolio Tracking Error*



Active Portfolio Return: Alpha

- Average annual performance
- Jensen's Alpha

Choice should be consistent to choice of TE definition…

IR (2/3) - TRACKING ERROR DEFINITIONS

$$\mu_P = \alpha + \beta \cdot \mu_B + \varepsilon$$

$$\sigma_P^2 = \beta^2 \cdot \sigma_B^2 + \sigma_\varepsilon^2 = \rho_{PB}^2 \cdot \frac{\sigma_P^2}{\sigma_B^2} \cdot \sigma_B^2 + \sigma_\varepsilon^2$$

with… $\beta = \frac{\sigma_{PB}}{\sigma_B^2} = \rho_{PB} \frac{\sigma_P}{\sigma_B}$

$$\sigma_P^2 = \rho_{PB}^2 \cdot \sigma_P^2 + \sigma_\varepsilon^2 \qquad \boxed{TE_P = \sigma_P \cdot \sqrt{1 - \rho_{PB}^2}} = \sigma_\varepsilon$$

…**Residual risk** = Risk uncorrelated with BM.

$$\boxed{TE_P = \sqrt{Var(r_P - r_B)}}$$

…**Standard deviation of performance**

$$\mu_P - \mu_B = \alpha + \beta \cdot \mu_B + \varepsilon - \mu_B = \alpha + (\beta - 1) \cdot \mu_B + \varepsilon$$

$$\sqrt{Var(\mu_P - \mu_B)} = \sqrt{(\beta - 1)^2 \cdot \sigma_B^2 + \sigma_\varepsilon^2}$$

→ For beta ≠ 1, the Stdev(perf) is always larger than residual risk

→ Stdev(perf) depends on benchmark volatility

IR (3/3) - APPLICATION

MEASUREMENT

- Best practice in CH: TE as annualized volatility of performance. Alpha as average annualized performance.
- Measurement of Alpha & TE with index or factor models makes IR dependent on model specification errors.

INTERPRETATION

- Summary statistic: Active return / active risk trade off, efficiency ratio
- Fundamental Law of Active Mgt:

  $IR_{ex\ ante} = IC \times BR$

  IC: Information Coefficient
  Corr(Forecast r, Actual r)

  BR: Breadth of strategy
  # of independent bets taken

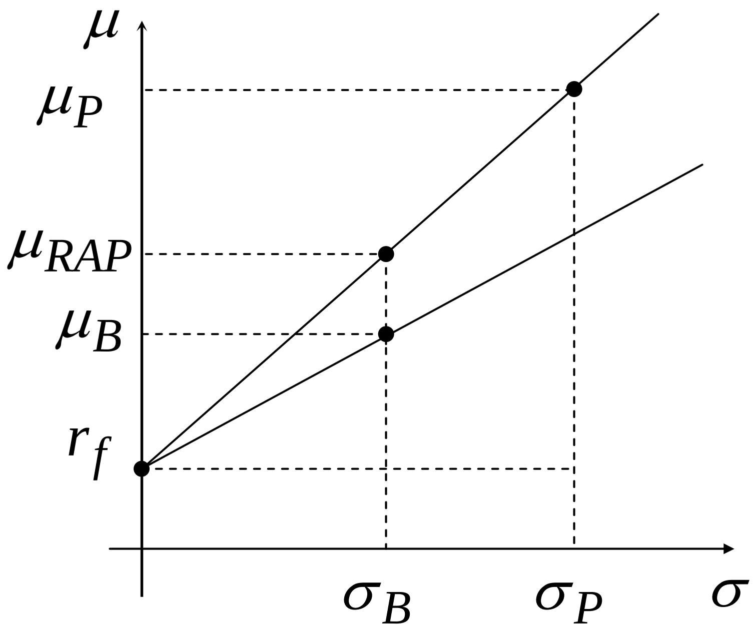
- Statistical hypothesis testing: Non-zero alpha signals

  t-Stat = IR * sqrt(T)

- Generally not consistent with MVO…

M MEASURES - M² (1/2)

$$\mu_{RAP} = \frac{\sigma_B}{\sigma_P}\left(\mu_P - r_f\right) + r_f$$



μ_{RAP} ...*Risk - Adjusted Return*

σ_P ...*Portfolio Volatility*

σ_B ...*Benchmark Volatility*

μ_f ...*Portfolio Return*

r_f ...*Riskfree Rate*

→ Performance is volatility-adjusted by leveraging the fund with risk-free-investments so that the resulting volatility equals the benchmark volatility.

$\frac{\sigma_B}{\sigma_P}$ ...*Leverage Factor d*

$$\mu_{RAP} = d \cdot \mu_P + (1 - d) \cdot r_f$$

M MEASURES - M^2 (2/2)

- The difference between M^2 can be interpreted intuitively: Unit of measurement is % $\rightarrow$ Risk expressed in units of return
- M^2 rankings are independent of the chosen benchmark (benchmark risk as a scaling factor)
- The M^2 measure is a transformed Sharpe Ratio and therefore consistent with MPT

$$\mu_{RAP} = \frac{\sigma_B}{\sigma_P}\left(\mu_P - r_f\right) + r_f = \sigma_B \cdot S + r_f$$

- M^2 ranking equals Sharpe Ratio ranking
- Drawback: Correlation risk (timing, selection) is neglected…

M MEASURES - M³ (1/2)

$$\mu_{CAP} = a \cdot \mu_P + b \cdot \mu_B + (1 - a - b) \cdot r_f$$

$$\overline{\rho}_{PB} = 1 - \frac{\overline{TE_{PB}}^2}{2 \cdot \sigma_B^2}$$

$$a = \frac{\sigma_B}{\sigma_P} \sqrt{\frac{(1 - \overline{\rho}_{PB}{}^2)}{(1 - \rho_{PB}{}^2)}}$$

$$b = \overline{\rho}_{PB} - \rho_{PB} \sqrt{\frac{(1 - \overline{\rho}_{PB}{}^2)}{(1 - \rho_{PB}{}^2)}}$$

→ M^3 cannot be illustrated graphically in an elegant way (three dimensions)

→ Performance is correlation-adjusted by leveraging the fund with active, passive and risk-free funds so that (1) the resulting volatility equals benchmark volatility and (2) the TE equals the Target TE

M MEASURES - M^3 (2/2)

- M^3 is 'volatility-risk- and-correlation-risk'-adjusted-performance
- M^3 rankings differ from M^2 and rankings
- If no target tracking error exists, a = 0 and M^3 will equal M^2
- M^3 can be used in a forward looking sense: It can provide ex ante guidance how to structure portfolios with TE restrictions (given the stability of distributional characteristics in the future)
- Drawback (of all RAP measures): Timing and selection activities are not decoupled.

RAPP (1/2)

…**R**isk-**A**djusted **P**erformance and **P**ositioning Index

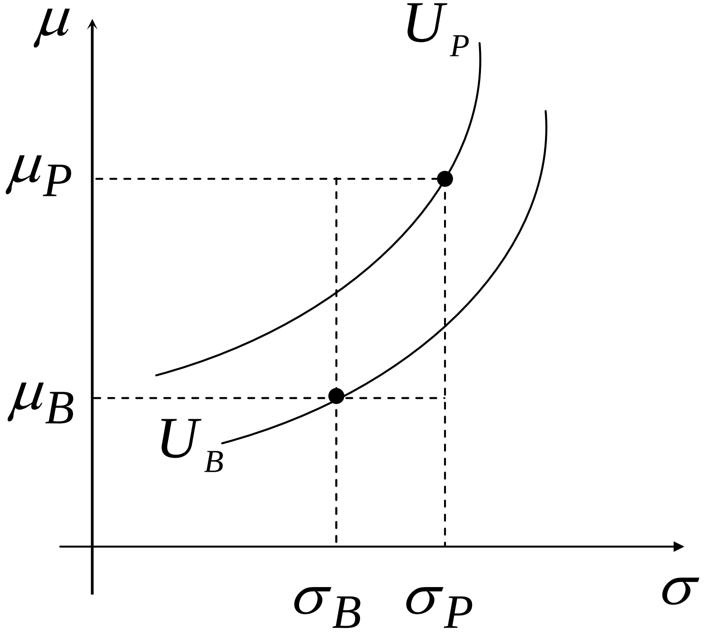
$$U(\mu_P,\sigma_P)\approx U(\mu_B,\sigma_B)+\frac{\partial U}{\partial \mu}d\mu+\frac{\partial U}{\partial \sigma}d\sigma$$

$$RAPP\equiv\frac{U(\mu_P,\sigma_P)-U(\mu_B,\sigma_B)}{\partial U/\partial\mu}\approx\alpha+\lambda\cdot TE$$

$$\lambda=\frac{\partial U/\partial\sigma}{\partial U/\partial\mu}\quad ...Risk\ Aversion$$

$$\alpha\approx d\mu$$

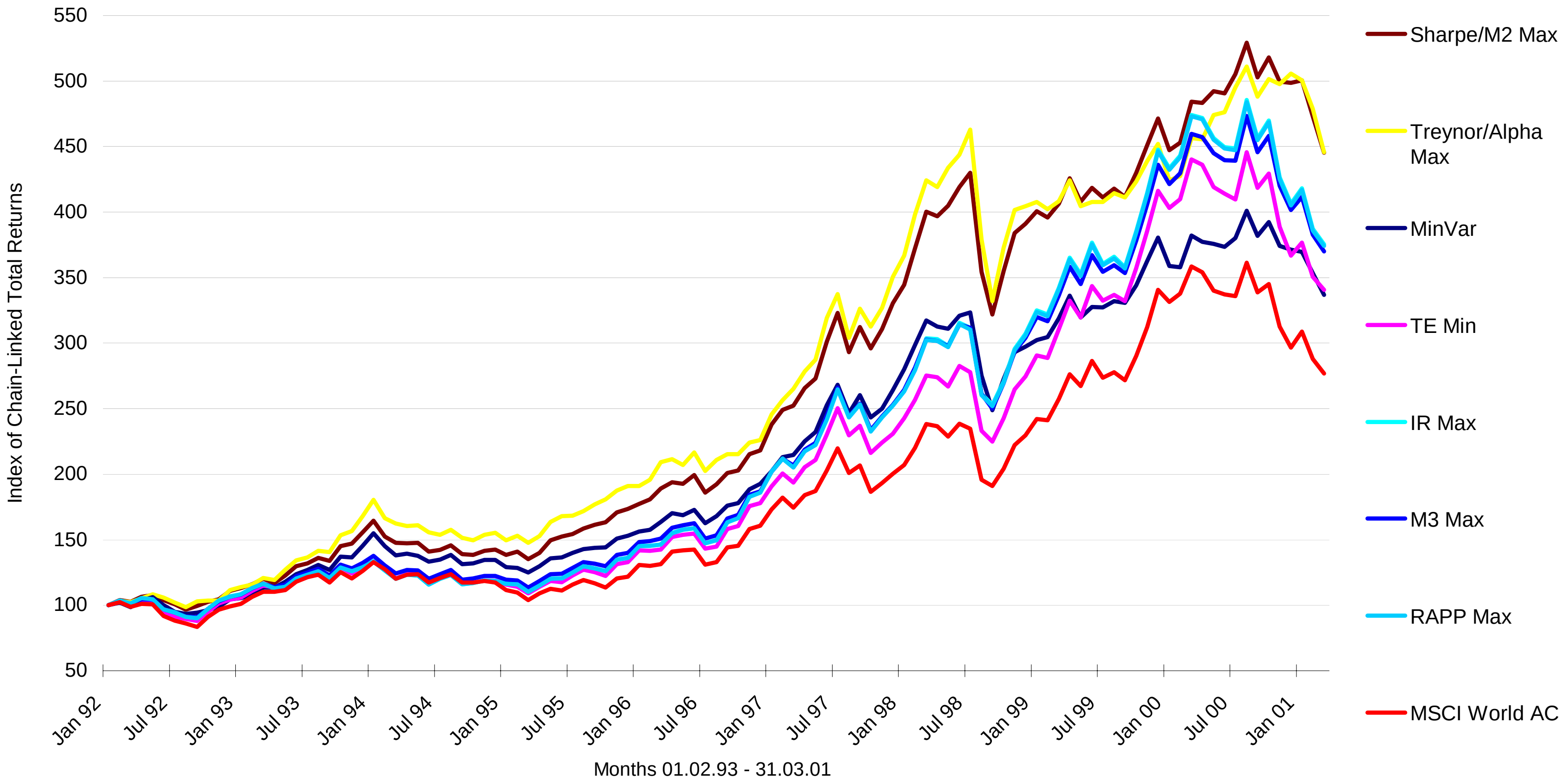
$$TE\approx d\sigma$$



RAPP (2/2)

- The RAPP concept is very flexible (TE targets, for example)
- Utility functions are considered at least problematic by many economists, especially in decision making under risk (‘Homo Oeconomicus’ debate, Behavioral Finance)
- To implement RAPP, the marginal utilities of parameters (risk aversion, for example) have to be quantified. RAPP ranking will depend on these marginal utilities.
- Aggregation across asset classes is achieved by measuring everything in terms of utilities. A new aggregation problem is introduced: aggregating client preferences.
- Non-financial aspects are neglected. Considering the importance of such factors: Is it worth developing and maintaining an internal RAP measure?

RELATIONSHIP BETWEEN MEASURES

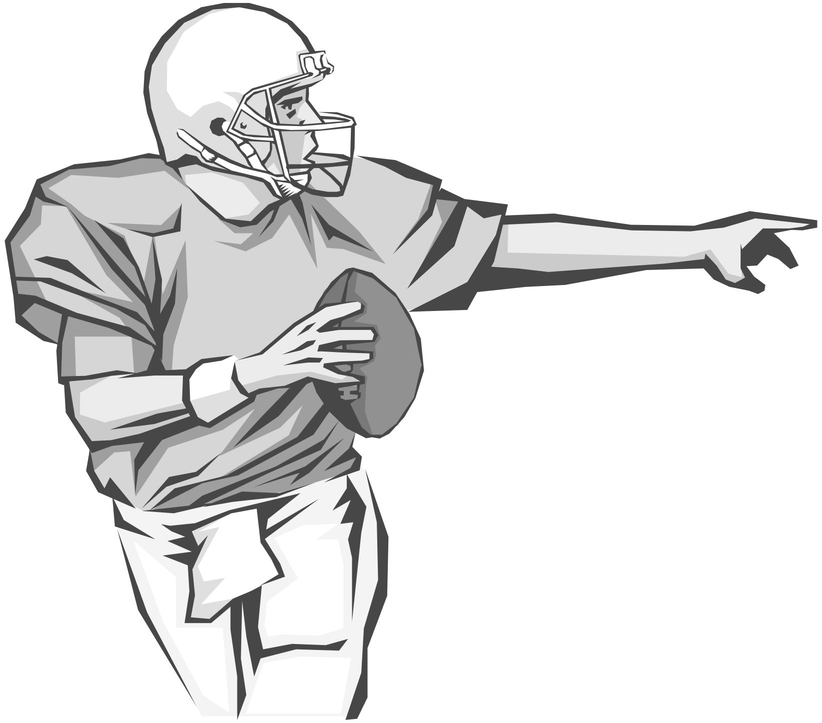


Markets: S&P 500, DJ Euro STOXX 50, SPI, MSCI Japan, FTSE 100

Observations:

- RAP strategies are highly correlated
- The ex ante / ex post choice of RAP targets creates significant incentives

DISCUSSION



IT'S YOUR TURN...